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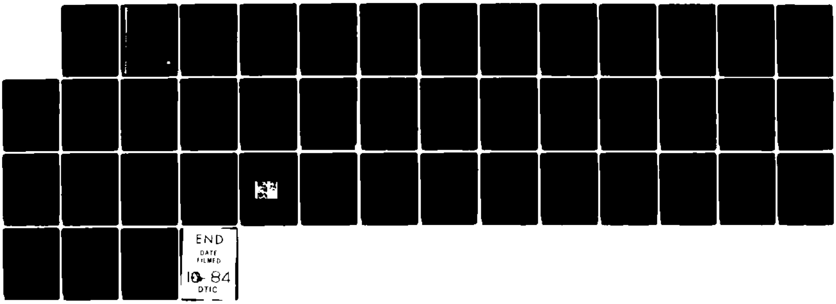

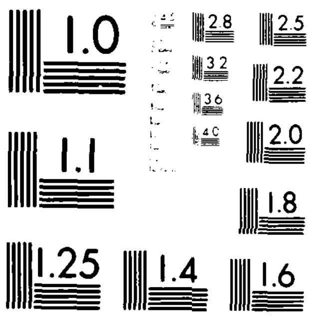
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FINAL TECHNICAL REPORT

DESIGN OF EDGE DETECTORS FOR REDUCED IMAGES

By

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The development of algorithms to extract informational features from imagery is an area of active research. These algorithms enable computerized devices to automatically locate and identify objects in the field of view of a sensor. An important Air Force application is automatic target identification and weapon guidance. Edges in an image contain much of the information necessary to classify objects.

This investigation has centered on finding methods for reducing an image so as to maximize the retention of edge information which was subsequently extracted. The Hotelling transform which reduces image data so as to minimize intensity mean-square error (IMSE) in the reconstructed image was also found to have significantly better edge retaining ability than simple averaging. The reconstructed edges were quantitatively compared to those in the original images using MSE and receiver operating characteristic based measures. One such measure used was the gradient mean-square error (CONTINUED)

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ITEM #19, ABSTRACT, CONTINUED: (GMSE). Both the reconstructed IMSE and GMSE using the Hotelling transform tend to decrease as the encoding block size increases. An equation relating GMSE to IMSE was developed.

For image gradient blocks that are independently reconstructed, the linear transformation matrix A that minimizes the reconstructed GMSE and in that sense maximizes edge retention was derived.

DESIGN OF EDGE DETECTORS FOR REDUCED IMAGES

1. INTRODUCTION

The development of algorithms to extract informational features from imagery is an area of active research. These algorithms enable computerized devices to automatically locate and identify objects in the field of view of a sensor. An important Air Force application is automatic target identification and weapon guidance.

Practical implementation of feature extracting algorithms is constrained by size, weight, power and data throughput requirements imposed by the particular application. These constraints limit the amount of computation that can be accomplished on the input image data. In order to allow more of the available computational time and hardware to be devoted to sophisticated and computationally intensive feature classification routines, it is desirable to minimize the preliminary feature extracting calculations.

One approach to the problem of reducing calculations is to map the original intensity image into a smaller image before applying feature extracting algorithms. Several such image data reducing techniques were implemented in [1], followed by standard (Roberts [2-4] and Kirsch [2,3]) edge detecting operations on the reduced images. Since edges in an image contain much of the information necessary to classify objects, edge detectability and edge quality were used in that work as measures of feature information loss. The outputs of the standard edge detectors were quantitatively evaluated to determine the effect of the image reducing techniques on edge content.

For the image reducing techniques implemented in [1], it is evident that the straightforward application of standard edge detectors to the reduced images does not fully extract the edge information that is available. This is demonstrated by the substantially better results achieved by applying the Kirsch operator to re-expanded versions of the reduced images. The re-expansion was accomplished by simply duplicating reduced image picture elements (pixels), so the information content would seem to be unaffected. The two-step sequence of re-expansion followed by Kirsch edge detection is equivalent to a new edge detection scheme operating on a reduced image and generating multiple output pixels for each input pixel. The re-expansion operation inherently incorporates knowledge of the original image size, adding information that was not available in the reduced image. However, the two-step sequence does not incorporate any knowledge of the reducing technique used.

The purpose of this research is to find techniques for extracting edges from reduced images based on knowledge of the specific image reducing techniques used. Incorporating knowledge of the image reducing techniques used should result in edge outputs that are more representative of the edge information available in reduced images.

The search for better ways to extract edges from reduced images leads naturally to consideration of how the image data should be reduced in the first place. As a result this investigation centers on finding methods for reducing an image so as to maximize the retention of edge information which is subsequently extracted. Since lower mean-square error (MSE) in the intensity domain was usually (with one exception) found to correspond to lower MSE in the edge domain in [1], The Hotelling transform [2,5] (also called the discrete Karhunen-Loeve transform [2,4]) is initially investigated. The Hotelling transform minimizes the intensity mean-square error (IMSE) between an original image and one that has been reconstructed by an inverse Hotelling transformation using partial transform coefficients (i.e. reduced image data). Subsequently a measure of edge loss called the gradient mean-square error (GMSE) based on the Roberts gradient is defined, and the relationship between GMSE and IMSE is derived. Finally, a linear transformation is introduced which reduces an image based on minimizing GMSE (and in that sense maximizing edge retention).

2. OBJECTIVE

The primary purpose of this research is to find techniques for extracting edges from reduced images based on knowledge of the specific image data reducing techniques used.

3. EDGE EVALUATION TECHNIQUES

Several techniques are used in subsequent sections of this report to compare edges in original and reconstructed images. Three original 256 by 256 images differing in edge content are used in this study. They are shown in Figure 1 and will be referred to as Image A (upper left), Image B (upper right), and Image C (lower left).

It is difficult to find edge evaluation techniques applicable to real-world imagery. This is due in part to the inability to clearly define the actual location of edges in an image and thus form a reliable basis for comparison. However, a paper by Kitchen and Rosenfeld [4] proposes an edge quality evaluation based on edge coherence incorporating connectedness, thinness, and gradient directions as seen by 3x3 neighborhoods. The specific implementation used here is described in [1]. In this implementation a Kirsch edge detector is applied to an image. Each pixel location in the resulting Kirsch image that has a Kirsch response equal to or greater than a selected threshold is assigned an edge coherence value

$$E = wC + (1-w)T \tag{3.1}$$

where C and T are measures of edge continuity and thinness, respectively, and w is a weighting factor. E, C, T, & w are constrained to lie in the interval [0,1]. In this study w = 0.8 as recommended in [6]. The average value of E for a selected threshold is used as an overall measure of image edge quality.

In order to determine the edge retaining abilities of an image data reducing technique some measure of the edge information lost in the transformation is needed. One such measure based on the Roberts edge detector [2-5] is obtained by calculating a gradient vector G at each pixel in the original and reconstructed images and determining the mean-square error between the original and reconstructed gradient vectors. This gradient mean-square error (GMSE) is used as a measure of edge information loss. For a pixel X(p,q) at image location (row,column)=(p,q) the specific gradient used was defined as

$$G=0.5*I*[(X(p,q-1) + X(p,q)) - (X(p-1,q-1) + X(p-1,q))] +0.5*J*[(X(p-1,q) + X(p,q)) - (X(p-1,q-1) + X(p,q-1))] \tag{3.2}$$

where I and J are unit vectors in the directions down and right, respectively, and * represents multiplication.

Two other MSE based measures were obtained by operating on original and reconstructed images using the Roberts (or Kirsch) edge detector and calculating the resulting MSE between the original and reconstructed edge images averaged over all pixel locations to obtain the Roberts (or Kirsch) MSE. A fourth MSE measure which will be referred to as the thresholded Kirsch MSE is obtained by calculating the Kirsch MSE averaged over only those pixel locations where the

Kirsch response is equal to or greater than a selected threshold.

The last edge comparison technique implemented uses an approach from signal theory which calculates the probabilities of edge detection PD and false alarm PF. These calculations use binary reference and test images generated by globally thresholding the edge detector output images. The resulting edge pixels are compared to give PD and PF where:

PD is the fraction of edge pixels in the reference image that are correctly classified as edge pixels in the test image;

and

PF is the fraction of non-edge pixels in the reference image that are incorrectly classified as edge pixels in the test image.

4. IMAGE DATA REDUCTION USING THE HOTELLING TRANSFORM

Since lower MSE in the intensity domain was found in [1] to strongly correlate with lower MSE in the edge domain, the effect of the Hotelling transform on edge retention is studied here. The Hotelling transform minimizes the IMSE between an original image and one that has been reconstructed by an inverse Hotelling transformation using partial transform coefficients. As in [1], this investigation concentrates on 4:1 data compression.

The Hotelling transform was implemented by partitioning an image into adjacent non-overlapping N by N blocks. Each N by N block is row scanned to form a vector X with N**2 elements, where ** represents exponentiation. Using notation similar to that in [5], a complete set of N**2 Hotelling transform coefficients forming a vector Y could be calculated using

$$Y = A(X - MX) \tag{4.1}$$

where the vector MX is the ensemble mean of the vector X averaged over all blocks in the image, and A is an N**2 by N**2 matrix whose rows are the eigenvectors of the image covariance matrix

$$CX = E\ \{(X - MX)(X - MX)'\} \tag{4.2}$$

where E is the expectation operator averaged over all blocks in the image and the prime (') indicates transposition. To achieve image data reduction ISAVE coefficients are generated (ISAVE < N**2) using an ISAVE by N**2 matrix A.

For a specified amount of data compression the Hotelling transform minimizes the IMSE in the reconstructed image by choosing the rows of A to be the ISAVE eigenvectors corresponding to the ISAVE largest eigenvalues of the covariance matrix CX. To achieve 4:1 image data reduction ISAVE = (N**2)/4 was used. For each N by N block a transform coefficient vector Y with dimension ISAVE is generated. Images are reconstructed a block at a time from the reduced image data using the inverse Hotelling transformation

$$XEST = A'Y + MX \tag{4.3}$$

where XEST is the reconstructed estimate of X.

For various values of N the test images were data compressed and reconstructed using the Hotelling transform. Then the evaluation techniques described in the previous section were applied to the reconstructed images.

For comparison purposes these same evaluation techniques were also applied to images which had been reduced by simply averaging 2 by 2 blocks and then reconstructed by pixel duplication. The results are shown in Tables 1 & 2. Table 1 values were calculated for all three test images. Table 2 values were calculated for Image A only using a threshold value of 99 which corresponds to a maximum average value of edge coherence E for the original Image A. The column labelled CDE in Table 1 will be defined in the next section.

As is well known, for any specified amount of compression the reconstructed IMSE using the Hotelling transform tends to decrease as the block size increases. This trend is evident in Table 1. The Hotelling transform accomplishes this by taking advantage of correlations that exist between neighboring pixels. As N gets larger than the maximum distance between pixels with significant correlation, a plot of IMSE versus N would tend to level off. From Table 1 there appears to still be significant correlation between pixels separated by a distance of 8 since IMSE is still dropping rapidly for N=8. The simple averaging of 2 by 2 blocks is unable to take advantage of correlations of pixels separated by distances greater than one. Therefore, as expected, the Hotelling transform offers substantially better IMSE performance than simple averaging. Notably, the edge evaluating measures shown in Tables 1 & 2 also demonstrate that the Hotelling transform has significantly better edge retaining ability than simple averaging.

A major obstacle to the use of the Hotelling transform in real-time encoding environments is the calculation of the eigenvectors of the block covariance matrix CX for each image. It may however be possible to generate a single Hotelling transform matrix A based on some selected covariance matrix that will retain edges satisfactorily over a wide variety of images. If necessary a small set of selectable A matrices might be stored for real-time use. To study the sensitivity of edge retention to the particular transformation matrix A used, the test images were reduced and reconstructed using Hotelling transformations based on each others covariance matrices. The resulting GMSE values are given in Table 3. Images B and C appear quite insensitive to the specific covariance matrix used. Image A, which contains the most edges, is the most sensitive to the covariance matrix selected. Covariance matrix selection based on the type of terrain being viewed should be practical.

5. RELATIONSHIP BETWEEN GMSE & IMSE

As noted in the previous section, there appears to be a strong correlation between IMSE and the edge domain measures. In this section the edge loss measure GMSE is found to be related to the IMSE by

$$GMSE = 2*(IMSE - CDE) \tag{5.1}$$

where CDE is the correlation of diagonal errors defined by

$$\begin{aligned} CDE=0.5*E\{&[X(p,q-1)-(XEST(p,q-1)]*[X(p-1,q)-XEST(p-1,q)] \\ &+ [X(p,q)-XEST(p,q)]*[X(p-1,q-1)-XEST(p-1,q-1)]\} \end{aligned} \tag{5.2}$$

where XEST is the reconstructed estimate of X, and E is the expectation operator. Here E may be thought of as a spatial operator that averages over all pixel locations (p,q) in an image, or equivalently it may be thought of as a combination of a spatial operator averaging over an N by N block of an image and an ensemble operator averaging over all such blocks in an image. Both viewpoints will be used in the sequel.

From its definition the CDE is seen to be a measure of the correlation of the reconstruction error between diagonal neighbors. For example, if a hypothetical transform resulted in a reconstructed image that was merely the original image decreased by 5 intensity units at all pixel locations (i.e. XEST(p,q) = X(p,q)-5 for all (p,q)), then diagonal errors would be totally correlated and the diagonal intensity differences upon which the gradient is based would be unaffected (as would all edge information). In such a hypothetical case IMSE=CDE=25 and GMSE=0. However, as seen in Table 1, the Hotelling transform consistently results in negative CDE values, causing GMSE to be greater than 2*IMSE.

Proof of Relationship:

The GMSE is the expected value of the square of the norm of the gradient error defined by

$$\text{GMSE} = E\,\{\|\ G(p,q) - \text{GEST}(p,q)\ \|^{**}2\} \tag{5.3}$$

where || || denotes the norm, GEST(p,q) represents the estimate of the gradient at location (p,q) obtained from a reconstructed image, and the expectation operator E is defined as described earlier. This may be written making the spatial averaging over an N by N block explicit and reducing E to a mere ensemble expectation as shown on the next page.

$$\text{GMSE} = \frac{1}{4*N**2} \sum_{p=1}^{N} \sum_{q=1}^{N} E \Big\{$$

$$\begin{aligned}
&[\ (X(p,q-1) - XEST(p,q-1)) \\
&\ + (X(p,q) - XEST(p,q)) \\
&\ - (X(p-1,q-1) - XEST(p-1,q-1)) \\
&\ - (X(p-1,q) - XEST(p-1,q))]**2 \\
+\ &[\ (X(p-1,q) - XEST(p-1,q)) \\
&\ + (X(p,q) - XEST(p,q)) \\
&\ - (X(p-1,q-1) - XEST(p-1,q-1)) \\
&\ - (X(p,q-1) - XEST(p,q-1))]**2 \Big\}
\end{aligned}$$

$$= \frac{1}{4*N**2} \sum_{p=1}^{N} \sum_{q=1}^{N} E \Big\{$$

$$\begin{aligned}
&2*[\ (X(p,q-1) - XEST(p,q-1))**2 \\
&\quad + (X(p,q) - XEST(p,q))**2 \\
&\quad + (X(p-1,q-1) - XEST(p-1,q-1))**2 \\
&\quad + (X(p-1,q) - XEST(p-1,q))**2] \\
&-4*[\ (X(p,q-1) - XEST(p,q-1)) \\
&\qquad *(X(p-1,q) - XEST(p-1,q)) \\
&\quad + (X(p,q)-XEST(p,q)) \\
&\qquad *(X(p-1,q-1)-XEST(p-1,q-1))]\ \Big\}
\end{aligned} \tag{5.4}$$

which reduces to

$$\text{GMSE} = 2*(\text{IMSE} - \text{CDE}) \tag{5.5}$$

Note that the gradient at pixel locations in the top row and left column of an N by N block depend on neighboring pixels above and to the left of the block. The above notation is based on the top row and left column of neighbors being designated row 0 and column 0, respectively.

Within minor roundoff errors this relationship is verified by Table 1. For example using N=4 on Image A gives CDE = -76 and IMSE = 371 for a calculated GMSE of

$$GMSE = 2*(371 + 76) = 894$$

which is reasonably close to the simulated value of 887.

6. A GMSE BASED LINEAR TRANSFORMATION

The gradients of an N by N block of pixel locations within an image form a gradient block which depends on the border pixels at the top and to the left of the N by N block. A linear transformation is introduced which individually encodes and data compresses overlapping (N+1) by (N+1) intensity blocks so as to minimize the resulting GMSE for the N by N gradients which are calculated for each reconstructed intensity block. This formulation allows each N by N gradient block to be independently reconstructed using the transform coefficients generated from its corresponding (N+1) by (N+1) intensity block. As with the Hotelling transform, the (N+1) by (N+1) pixels in each intensity block are row scanned to form a vector with (N+1)**2 elements, where ** represents exponentiation. A complete set of (N+1)**2 transform coefficients could be obtained by multiplying this vector by an (N+1)**2 by (N+1)**2 matrix A. To achieve data compression ISAVE coefficients are generated (ISAVE < (N+1)**2) using an ISAVE by (N+1)**2 matrix A. To minimize IMSE the Hotelling transform chooses the rows of A to be eigenvectors of the image block covariance matrix.

To determine the matrix A that minimizes GMSE a set of summations describing the GMSE as a function of the matrix A and block covariance matrix was derived. (This equation is derived at the end of this section and implemented in the attached Fortran 77 Subroutine DGMSE.) For a given block size, block covariance matrix, and specified data compression, the matrix A that minimizes GMSE was found numerically using a steepest descent algorithm based on an article by Fletcher and Powell [7]. This algorithm required the derivation of an equation describing the gradient of the function GMSE. (This equation is implemented in the attached Subroutine DGGMSE.)

The eigenvectors used by the Hotelling transform were used to form an initial guess for the matrix A that would minimize GMSE. The optimal matrix A was then calculated using the Fletcher-Powell search technique. For several combinations of N and ISAVE, Table 4 compares the resulting GMSE using the Hotelling eigenvector matrix to the GMSE achieved using the optimal matrix A. The theoretical values were obtained using the GMSE equation implemented in Subroutine DGMSE. The simulated values were obtained by actually transforming, data compressing, intensity reconstructing, and independently calculating the gradients in reconstructed overlapping (N+1) by (N+1) intensity blocks of Image A. Compared to the Hotelling transform, the optimal matrix A resulted in 18% to 52% lower GMSE in the cases studied.

Calculation of the optimal matrix A using the iterative

Fletcher-Powell algorithm is quite computationally intensive. For N=6 and ISAVE=9, a moderate block size, the matrix A consists of 9*(6+1)**2 = 441 elements. The Fletcher-Powell algorithm seeks to find the resulting 441-dimensional vector that minimizes GMSE. For this particular example 708 calls to the search routine described in [7] were needed, consuming approximately 200 hours of CPU time on a Data General MV 10000 computer. As with the Hotelling transform discussed in Section 4, this disadvantage may be overcome by storing a small set of selectable pre-calculated A matrices for real-time use.

The optimal matrix A described above minimizes the GMSE of gradient blocks in an image that are reconstructed independently. Similarly an optimal matrix A calculated for an entire image (based on an ensemble covariance matrix) without partitioning the image into blocks would achieve minimum GMSE over all linear transformations (although calculating A for entire images would be computationally prohibitive using iterative search techniques such as the Fletcher-Powell algorithm). However the independent reconstruction of adjacent gradient blocks does not allow correlations between adjacent blocks to be exploited. Using the Hotelling transform on non-overlapping N by N intensity blocks to data compress and reconstruct an entire intensity image, followed by gradient calculations on the reconstructed intensity image takes advantage of block-to-block correlations. Table 5 compares the GMSE for dependently reconstructed gradient blocks obtained using the Hotelling transform in this fashion to the GMSE obtained using the optimal matrix A which independently reconstructs each block. For the combinations of N and ISAVE shown in Table 5, adjacent block correlations cause the GMSE of independently reconstructed gradient blocks to be inferior. For the larger block sizes shown in Table 5 the advantage of adjacent block correlations seems to diminish. For block sizes greater than N=6 it is not yet known which method will result in lower GMSE. However, as stated above, if the block size increases to encompass the entire image, the optimal matrix A would achieve minimum GMSE while the relative performance of the Hotelling transform is unknown.

Derivation of GMSE as a function of A and CX:

Equation 5.4 shows that the GMSE is related to the expected value of a function of original and reconstructed block pixel intensity values X and XEST, respectively. At this point an expression is derived relating GMSE to the elements of the ISAVE by (N+1)**2 transform matrix A and the elements of the (N+1)**2 by (N+1)**2 block covariance matrix CX. As with the Hotelling transform in Equation 4.1, the ensemble mean for each block pixel is subtracted prior to multiplication by the transform matrix A. Therefore the following derivation can be simplified by assuming the image block values X have zero mean which reduces Equations 4.1-4.3 to

$$Y = AX \tag{6.1}$$

$$CX = E\{XX'\} \tag{6.2}$$

$$XEST = A'Y \tag{6.3}$$

For ease of presentation the following derivation is based on N=4 so that 5 by 5 overlapping blocks are transformed. The extension of the results to arbitrary values of N is trivial. For N=4, Equation 6.1 may be written as

$$Y(i) = \sum_{m=o}^{N} \sum_{n=o}^{N} A(i,5m+n+1)*X(m,n) \tag{6.4}$$

where Y(i) is the ith element of Y for i from 1 to 25. Then Equation 6.3 becomes

$$\begin{aligned} XEST(p,q) &= \sum_{i=1}^{ISAVE} A(i,5p+p+1)*Y(i) \\ &= \sum_{i=1}^{ISAVE} A(i,5p+q+1) \\ &\qquad * \sum_{m=o}^{N} \sum_{n=o}^{N} A(i,5m+n+1)*X(m,n) \end{aligned} \tag{6.5}$$

for p and q from 0 through 4. Expanding the quadratic terms in Equation 5.4 and taking the expectation of each resulting product we obtain terms of the following forms:

$$E\{X(p,q)X(r,s)\} = CX(5p+q+1,5r+s+1) \tag{6.6}$$

$$E\{X(p,q)XEST(r,s)\} =$$

$$= \sum_{i=1}^{ISAVE} A(i,5r+s+1)$$

$$* \sum_{m=o}^{N} \sum_{n=o}^{N} A(i,5m+n+1) * E\{X(p,q)X(m,n)\}$$

$$= \sum_{i=1}^{ISAVE} \sum_{m=o}^{N} \sum_{n=o}^{N} A(i,5r+s+1)A(i,5m+n+1)$$

$$* CX(5p+q+1,5m+n+1) \tag{6.7}$$

$$E\{XEST(p,q)XEST(r,s)\} =$$

$$= E\{ \sum_{i=1}^{ISAVE} \sum_{m=o}^{N} \sum_{n=o}^{N} A(i,5p+q+1)A(i,5m+n+1)X(m,n)$$

$$* \sum_{i'=1}^{ISAVE} \sum_{m'=o}^{N} \sum_{n'=o}^{N} A(i',5r+s+1)A(i',5m'+n'+1)$$

$$* X(m',n') \}$$

$$= \sum_{i=1}^{ISAVE} \sum_{m=o}^{N} \sum_{n=o}^{N} \sum_{i'=1}^{ISAVE} \sum_{m'=o}^{N} \sum_{n'=o}^{N} \Big\{$$

$$A(i,5p+q+1)A(i,5m+n+1)A(i',5r+s+1)A(i',5m'+n'+1)$$

$$* CX(5m+n+1,5m'+n'+1)\Big\} \tag{6.8}$$

Using Equations 6.6 through 6.8 enables us to now rewrite

Equation 5.4 expressing GMSE as a function of matrices A and CX as shown on the next page.

$$
\begin{aligned}
GMSE = \frac{1}{4*N**2} \sum_{p=1}^{N} \sum_{q=1}^{N} \Bigg\{ \\
& 2[CX(5p+q+1,5p+q+1)+CX(5p+q,5p+q) \\
& \quad +CX(5(p-1)+q,5(p-1)+q)+CX(5(p-1)+q+1,5(p-1)+q+1)] \\
& -4[CX(5p+q,5(p-1)+q+1)+CX(5p+q+1,5(p-1)+q)] \\
& -4 \sum_{i=1}^{ISAVE} \sum_{m=o}^{N} \sum_{n=o}^{N} A(i,5m+n+1) \{ \\
& \quad A(i,5p+q+1)[CX(5p+q+1,5m+n+1) \\
& \qquad -CX(5(p-1)+q,5m+n+1)] \\
& \quad +A(i,5p+q)[CX(5p+q,5m+n+1) \\
& \qquad -CX(5(p-1)+q+1,5m+n+1)] \\
& \quad +A(i,5(p-1)+q)[CX(5(p-1)+q,5m+n+1) \\
& \qquad -CX(5p+q+1,5m+n+1)] \\
& \quad +A(i,5(p-1)+q+1)[CX(5(p-1)+q+1,5m+n+1) \\
& \qquad -CX(5p+q,5m+n+1)] \\
& \quad -0.5 \sum_{i'=1}^{ISAVE} \sum_{m'=o}^{N} \sum_{n'=o}^{N} \\
& \qquad A(i',5m'+n'+1)CX(5m+n+1,5m'+n'+1) \\
& \qquad *[A(i,5p+q+1)A(i',5p+q+1)+A(i,5p+q)A(i',5p+q) \\
& \qquad +A(i,5(p-1)+q)A(i',5(p-1)+q) \\
& \qquad +A(i,5(p-1)+q+1)A(i',5(p-1)+q+1) \\
& \qquad -2*A(i,5p+q)A(i',5(p-1)+q+1) \\
& \qquad -2*A(i,5p+q+1)A(i',5(p-1)+q)] \} \Bigg\} \qquad (6.9)
\end{aligned}
$$

Equation 6.9, derived for N=4, can be generalized by replacing the numeral 5 in the CX and A matrix indices by (N+1). This generalized expression is programmed in the attached Subroutine DGMSE wherein the matrix A described here is row scanned to form a vector A with dimension ISAVE*(N+1)**2.

7. CONCLUSIONS AND RECOMMENDATIONS

The Hotelling transform which reduces image data so as to minimize intensity mean-square error (IMSE) in the reconstructed image was also found to have significantly better edge retaining ability than simple averaging. The reconstructed edges were quantitatively compared to those in the original images using the MSE based and receiver operating characteristic (PD and PF) measures described in Section 3. One such measure used was the gradient mean-square error (GMSE). Both the reconstructed IMSE and GMSE using the Hotelling transform tend to decrease as the encoding block size increases. An equation relating GMSE to IMSE was developed in Section 5.

For image gradient blocks that are independently reconstructed, Section 6 derives the linear transformation matrix A that minimizes the reconstructed GMSE, and in that sense maximizes edge retention. Calculation of the optimal matrix A is quite computationally intensive. The largest block size for which the matrix A was calculated was for overlapping 7 by 7 intensity blocks (N=6). In this case the GMSE obtained using the Hotelling transform on overlapping 7 by 7 intensity blocks to independently reconstruct 6 by 6 gradient blocks of Image A was 1042, 30% higher than the GMSE=801 obtained using the optimal matrix A. However, independent reconstruction of gradient blocks does not allow correlations between adjacent blocks to be exploited.

Dependent gradient block reconstruction using the Hotelling transform on non-overlapping 6 by 6 intensity blocks, reconstructing the resulting intensity image, and then applying the gradient operator resulted in GMSE=729, 9% lower than independent reconstruction using the optimal matrix A. Apparently at this block size adjacent block correlation plays a dominant role in determining the reconstructed GMSE.

This research has demonstrated that the edge information retained in a data compressed image can best be extracted by using knowledge of the image data reduction technique used. An optimal system design should include selecting the image reducing technique based on the reconstructed end product desired. For example, if the desired end product is a gradient image, the intensity image should be reduced so as to minimize some measure of the reconstructed gradient error such as GMSE.

Suggested areas for future research include: Simulations using larger block sizes comparing independent and dependent gradient block reconstruction errors.

Development of a transform which minimizes GMSE while including the effects of adjacent block correlation.

Development of transforms to minimize Kirsch or Sobel edge errors.

Development of transforms for maximum edge retention based on specific image covariance models.

Determine edge retaining abilities of other non-image dependent transforms.

Figure 1. Test Images A, B, & C

Table 1. MSE Based Edge Comparison of Averaging vs Hotelling Transform

	IMSE	GMSE	ROBERTS MSE	KIRSCH MSE	CDE
Image A:					
Averaging:	524	1367	1393	520	-164
Hotelling N=4:	371	887	760	366	-76
6:	305	729	641	289	-63
8:	279	704	619	261	-78
Image B:					
Averaging:	54	146	145	36	-19
Hotelling N=4:	34	84	70	29	-8
6:	26	62	54	21	-5
8:	23	54	48	18	-4
Image C:					
Averaging:	85	226	224	59	-29
Hotelling N=4:	55	134	114	50	-13
6:	43	102	92	39	-8
8:	38	91	81	34	-8

Table 2. Kirsch Based Edge Comparison of Averaging vs Hotelling Transform

	THRESHOLDED KIRSCH MSE	PD	PF	E
Image A:				
Original:	0	1	0	.780
Averaging:	231	.469	.0187	.747
Hotelling N=4:	184	.475	.0071	.791
6:	138	.541	.0061	.784
8:	116	.569	.0053	.784

PD = Probability of Detection
PF = Probability of False Alarm
E = Average Edge Coherence

Table 3. Sensitivity of GMSE to Image Block Covariance Matrix CX

N = 4; ISAVE = 4

IMAGE TRANSFORMED	IMAGE CX USED	GMSE
A	A	887
A	B	1090
A	C	1098
B	A	87
B	B	84
B	C	85
C	A	139
C	B	136
C	C	134

Table 4. GMSE of Independently Reconstructed Gradient Blocks of Image A

	THEORETICAL GMSE	SIMULATED GMSE
N=2 ISAVE=2:		
Hotelling:	1391	1374
Optimal:	914	902
N=3 ISAVE=4:		
Hotelling:	979	965
Optimal:	727	714
N=4 ISAVE=4:		
Hotelling:	1242	1227
Optimal:	1056	1042
N=6 ISAVE=9:		
Hotelling:	1042	1030
Optimal:	801	790

Table 5. Independent versus Dependent Gradient Block Reconstruction

	GMSE
N=2 ISAVE=2:	
Independent:	902
Dependent:	620
N=3 ISAVE=4:	
Independent:	714
Dependent:	424
N=4 ISAVE=4:	
Independent:	1042
Dependent:	888
N=6 ISAVE=9:	
Independent:	790
Dependent:	729

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APPENDIX

of

Fortran Subroutines

```
      SUBROUTINE DGMSE(CX,A,N,ISAVE,IASIZE,ICXSIZ,RMSE)
C     CALCULATE THEORETICAL GRADIENT MSE TRANSFORMING (N+1)X(N+1) BLOCKS
C     AND SAVING ISAVE COEFFICIENTS.
C     THE VECTOR A IN THIS SUBROUTINE IS A ROW SCANNED VERSION OF
C     THE ORIGINAL ISAVE X (N+1) MATRIX A.
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DOUBLE PRECISION CX(ICXSIZ,ICXSIZ),A(IASIZE)
      INTEGER P,Q,U,V,C,D,E,F
      N1=N+1
      N1SQ=N1**2
      RMSE=0.D0
      DO 10 P=1,N
      DO 10 Q=1,N
      C=N1*(P-1)+Q
      D=C+1
      E=N1*P+Q
      F=E+1
      SUM1=2.D0*(CX(F,F)+CX(E,E)+CX(C,
     1C)+CX(D,D))-4.D0*
     1(CX(E,D)+CX(F,C))
      SUM2=0.D0
      SUM3=0.D0
      DO 20 I=1,ISAVE
      U=(I-1)*N1SQ
      DO 20 M1=1,N+1
      DO 20 NN1=1,N+1
      M=M1-1
      L=NN1-1
      MCX=N1*M+L+1
      SUM2=SUM2+
     1A(U+F)*(CX(F,MCX)-CX(C,MCX))+
     1A(U+E)*(CX(E,MCX)-CX(D,MCX))+
     1A(U+C)*(CX(C,MCX)-CX(F,MCX))+
     1A(U+D)*(CX(D,MCX)-CX(E,MCX))
      DO 30 J=1,ISAVE
      V=(J-1)*N1SQ
      DO 30 MP1=1,N+1
      DO 30 NP1=1,N+1
      MP=MP1-1
      NP=NP1-1
      MPCX=N1*MP+NP+1
      SUM2=SUM2-0.5D0*(A(U+F)*A(V+F)+
     1A(U+E)*A(V+E)+A(U+C)*A(V+C)+
     1A(U+D)*A(V+D)-2.D0*
     1A(U+E)*A(V+D)-2.D0*A(U+F)*A(V+C))*
     1A(V+MPCX)*CX(MPCX,MCX)
   30 CONTINUE
      SUM3=SUM3+SUM2*A(U+MCX)
   20 SUM2=0.D0
   10 RMSE=RMSE+SUM1-4.D0*SUM3
      RMSE=RMSE/(4.D0*DBLE(N**2))
      RETURN
      END
```

```
      SUBROUTINE DGGMSE(CX,A,G,N,ISAVE,IASIZE,ICXSIZ)
C     CALCULATES GRADIENT OF GMSE
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DOUBLE PRECISION G(IASIZE),A(IASIZE),CX(ICXSIZ,ICXSIZ)
      INTEGER X,Y,P,Q,C,D,E,F,U,V
      N1=N+1
      N1SQ=N1**2
      X=1
      Y=0
7     Y=Y+1
      IF(Y.GT.N1SQ)X=X+1
      IF(X.GT.ISAVE)RETURN
      IF(Y.GT.N1SQ)Y=1
      U=(X-1)*N1SQ
      IC=0
      ID=0
      IE=0
      IF=0
      SUM=0.D0
      DO 10 P=1,N
      DO 10 Q=1,N
      C=N1*(P-1)+Q
      D=C+1
      E=N1*P+Q
      F=E+1
      SUMJMN=0.D0
      DO 15 J=1,ISAVE
      V=(J-1)*N1SQ
      DO 15 MP=0,N
      DO 15 NP=0,N
      MPCX=N1*MP+NP+1
15    SUMJMN=SUMJMN+(A(U+C)*A(V+C)+A(U+D)*A(V+D)+A(U+E)*A(V+E)+
     1A(U+F)*A(V+F)-2.D0*(A(U+E)*A(V+D)+A(U+F)*A(V+C)))*
     1A(V+MPCX)*CX(MPCX,Y)
10    SUM=SUM-0.5D0*SUMJMN+A(U+C)*(CX(C,Y)-CX(F,Y))+
     1A(U+D)*(CX(D,Y)-CX(E,Y))+A(U+E)*(CX(E,Y)-CX(D,Y))+
     1A(U+F)*(CX(F,Y)-CX(C,Y))
      SUMMN=0.D0
      DO 20 M=0,N
      DO 20 L=0,N
      SUMCX=0.D0
      JCX=N1*M+L+1
      ICNT=0
      JCNT=0
      DO 25 JY=1,N
26    ICNT=ICNT+1
      JCNT=JCNT+1
      IF(ICNT.GT.N)GO TO 25
      IF(Y.EQ.JCNT)GO TO 30
      GO TO 26
```

```
25  ICNT=0
    GO TO 35
30  IC=1
    SUMCX=SUMCX+CX(Y,JCX)-CX(Y+N1+1,JCX)
35  ICNT=N1+1
    JCNT=N*N1+1
    DO 45 JY=1,N
46  ICNT=ICNT-1
    JCNT=JCNT-1
    IF(ICNT.EQ.1)GO TO 45
    IF(Y.EQ.JCNT)GO TO 50
    GO TO 46
45  ICNT=N1+1
    GO TO 55
50  ID=1
    SUMCX=SUMCX+CX(Y,JCX)-CX(Y+N1-1,JCX)
55  ICNT=0
    JCNT=N1
    DO 65 JY=1,N
66  ICNT=ICNT+1
    JCNT=JCNT+1
    IF(ICNT.GT.N)GO TO 65
    IF(Y.EQ.JCNT)GO TO 70
    GO TO 66
65  ICNT=0
    GO TO 75
70  IE=1
    SUMCX=SUMCX+CX(Y,JCX)-CX(Y-N1+1,JCX)
75  ICNT=N1+1
    JCNT=N1SQ+1
    DO 85 JY=1,N
86  ICNT=ICNT-1
    JCNT=JCNT-1
    IF(ICNT.EQ.1)GO TO 85
    IF(Y.EQ.JCNT)GO TO 90
    GO TO 86
85  ICNT=N1+1
    GO TO 20
90  IF=1
    SUMCX=SUMCX+CX(Y,JCX)-CX(Y-N1-1,JCX)
20  SUMMN=SUMMN+A(U+JCX)*SUMCX
    SUM=SUM+SUMMN
    SUMIMN=0.D0
    DO 100 I=1,ISAVE
    V=(I-1)*N1SQ
    DO 100 M=0,N
    DO 100 L=0,N
    SUMPQ=0.D0
    JCX=N1*M+L+1
    DO 110 P=1,N
```

```
      DO 110 Q=1,N
      C=N1*(P-1)+Q
      D=C+1
      E=N1*P+Q
      F=E+1
110   SUMPQ=SUMPQ+CX(Y,JCX)*(
     1A(V+C)*A(U+C)+A(V+D)*A(U+D)+A(V+E)*A(U+E)+A(V+F)*A(U+F)-2.DO*
     1(A(V+E)*A(U+D)+A(V+F)*A(U+C)))
100   SUMIMN=SUMIMN+A(V+JCX)*SUMPQ
      SUM=SUM-0.5DO*SUMIMN
      SUMMNP=0.DO
      ACNT=DBLE(IC+ID+IE+IF)
      DO 200 M=0,N
      DO 200 L=0,N
      DO 200 MP=0,N
      DO 200 NP=0,N
      SUMA=0.DO
      MCX=N1*M+L+1
      MPCX=N1*MP+NP+1
      SUMI=0.DO
      DO 210 I=1,ISAVE
210   SUMI=SUMI+A((I-1)*N1SQ+Y)*A((I-1)*N1SQ+MCX)
      SUMA=SUMA+A(U+MPCX)*SUMI
      SUMJ=0.DO
      DO 220 J=1,ISAVE
220   SUMJ=SUMJ+A((J-1)*N1SQ+Y)*A((J-1)*N1SQ+MPCX)
      SUMA=SUMA+A(U+MCX)*SUMJ
      SUMA=SUMA*ACNT
      IF(IC.EQ.0)GO TO 259
      SUMI=0.DO
      DO 250 I=1,ISAVE
250   SUMI=SUMI+A((I-1)*N1SQ+Y+N1+1)*A((I-1)*N1SQ+MCX)
      SUMA=SUMA-2.DO*A(U+MPCX)*SUMI
259   IF(ID.EQ.0)GO TO 269
      SUMI=0.DO
      DO 260 I=1,ISAVE
260   SUMI=SUMI+A((I-1)*N1SQ+Y+N1-1)*A((I-1)*N1SQ+MCX)
      SUMA=SUMA-2.DO*A(U+MPCX)*SUMI
269   IF(IE.EQ.0)GO TO 279
      SUMJ=0.DO
      DO 270 J=1,ISAVE
270   SUMJ=SUMJ+A((J-1)*N1SQ+Y-N1+1)*A((J-1)*N1SQ+MPCX)
      SUMA=SUMA-2.DO*A(U+MCX)*SUMJ
279   IF(IF.EQ.0)GO TO 200
      SUMJ=0.DO
      DO 280 J=1,ISAVE
280   SUMJ=SUMJ+A((J-1)*N1SQ+Y-N1-1)*A((J-1)*N1SQ+MPCX)
      SUMA=SUMA-2.DO*A(U+MCX)*SUMJ
200   SUMMNP=SUMMNP+SUMA*CX(MPCX,MCX)
      SUM=SUM-0.5DO*SUMMNP
```

```
G(U+Y)=-1.D0*SUM/DBLE(N**2)
GO TO 7
END
```

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